

Hedging Portfolios with Real Assets

Which real assets can help protect financial portfolios?

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Investors are always searching for assets that are negatively correlated with common portfolios of stocks and bonds and that provide reasonable average returns. One broad group of such assets might be termed real assets — assets that are at least partially hedged against inflation, in that they tend to increase in price in response to an inflation shock.

Real assets ought to be valuable for portfolio diversification because widely diversified portfolios of stocks and bonds are strongly negatively correlated with inflation.¹ Thus, real assets are important partly because they might provide a kind of inflation insurance for other assets in the portfolio.

Although this is conventional wisdom, there is surprisingly little evidence on how investors ought to choose among real assets and how effectively different real assets protect financial portfolios.² For example, it is often argued that real estate helps protect returns on financial portfolios, but the properties of real estate are rarely compared with those of other real assets, such as commodity-linked equities or CPI-linked bonds. Commodities themselves are also often thought of as potential hedges, yet there is little discussion of whether commodity futures and commodity-linked equities are more appropriate.

This article examines the properties of a variety of asset classes that might broadly be thought of as “real” assets. It looks at how closely correlated these classes are with inflation, as well as how effectively they help insure major financial asset classes against adverse

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shocks. It also examines how inflation hedges might be combined in a portfolio.

For example, can holdings of real estate fully insure a stock/bond portfolio against inflation and other adverse shocks? Can holdings of commodities do a better job? What combinations of commodities, commodity-based equities, real estate, and CPI-linked bills are best for reducing the risk of equity and bond portfolios? As we will see, the answers to these questions can help guide the use of real assets as hedging vehicles.

We find evidence that oil and production-weighted indexes of commodity futures are the most potent hedges of financial portfolios. Direct investments in real estate are themselves hedged against inflation, but they are generally not good at hedging the rest of a portfolio. Moreover, certain commodity indexes and oil actually appear to be more powerful hedging vehicles than even inflation itself. For example, stocks and bonds respond negatively to commodity price increases even when such shocks are not related to measures of CPI inflation (an example would be the behavior of asset and consumer prices around the Kuwait war).

These results suggest two main conclusions. First, risk-averse investors will wish to bear a positive amount of commodity risk even when commodity futures pay expected returns that are lower than returns of other major asset classes. Indeed, to the extent that producers sell off oil exposures in futures and forward markets, equilibrium reasoning suggests that investors will want to buy those oil exposures as part of their diversified portfolios.

Second, the argument is weak for holding individual commodities, such as gold, or other real assets, such as real estate, on the basis of their hedging properties. A sounder argument would be that gold and real estate provide little hedging potential, but have attractive expected returns.

WHAT ARE REAL ASSETS?

Real assets are those that tend to increase in nominal value in the face of inflation. One easy way to identify such assets is by process of elimination. For example, the nominal payment stream from domestic fixed-income instruments (e.g., bonds) cannot rise with unexpected inflation. It follows that bonds cannot be real assets.

Foreign bonds (both hedged and unhedged

against currency fluctuations) are also unlikely to retain their real value when there is domestic inflation. This is because inflation shocks in the major industrialized countries are highly positively correlated with one another.

Assets such as stocks are more complex. On the one hand, diversified stock portfolios are claims against real physical and intangible assets, so it might seem that stock returns ought to be positively related to measures of unexpected inflation. Inflation surprises, however, can be bad news for stocks if they are correlated with adverse "real" shocks, such as declines in factor productivity or usage.

The empirical reality seems to favor the latter explanation: Not only do domestic stocks fail to keep up with inflation at horizons up to one year, but their nominal prices also actually *fall* when inflation unexpectedly rises. The negative response of domestic stocks to inflation carries over to foreign stocks as well; that is, foreign stock returns (regardless of whether they are hedged against unexpected currency movements) are negatively correlated with U.S. inflation.³

Even though broad portfolios of stocks may not behave as real assets, it seems reasonable to think that targeted industry stocks might do better. For example, the stocks of companies that own and/or manage primarily real estate assets might be expected to help hedge against inflation, according to the logic that real estate is itself a real asset. Thus, an investor looking to hedge inflation might consider direct holdings of real estate or the securitized equivalent, Real Estate Investment Trusts (REITs), which trade in the stock market. A second group of stocks that might behave more like real assets are commodity-producing firms, again on the logic that the values of the commodities they produce respond sharply to inflation. Thus, we might expect stocks of real estate-holding and commodity-producing firms to behave like real assets.

Of course, for hedging inflation the best hedge would be a CPI-linked bill or bond. The federal governments of the U.K. and Canada issue such bonds, as do those of a number of developing countries. While no such instrument exists in the U.S., its unexpected returns can be simulated by estimating unexpected U.S. inflation.

That is, unexpected inflation approximates the unexpected return on a zero-coupon CPI-linked instrument, with duration equal to the horizon over which unexpected inflation is measured. We therefore

use unexpected inflation as a proxy for the returns on a hypothetical CPI-linked bill. This will give us a benchmark against which to compare the performance of other hedging vehicles.

HEDGING EXPOSURES TO REAL ASSETS

Underlying any discussion of hedging policy is the notion of *exposure*. The exposure of an asset class to a real asset is the expected percent change in value of the asset per 1% change in the value of the real asset.⁴ For example, if a stock has an exposure of -0.5 to commodity inflation, then an unexpected 1% increase in commodity prices will on average decrease the value of the stock by 0.5%.

Conveniently, exposures are additive: The exposure of a portfolio of assets is just the weighted average of the exposures of the individual portfolio components. Thus, if stocks and bonds have inflation exposures of -0.1 and -0.7, respectively, the inflation exposure of an equally weighted stock/bond portfolio will be -0.4.

How does one optimally hedge a portfolio using real assets? Suppose that an investor holds a preexisting portfolio of risky stocks and/or bonds, and wishes to acquire exposure to real assets. To do so without selling any part of the preexisting portfolio, the investor can sell T-bills to purchase real assets. (A swap or futures contract linked to the real asset accomplishes the same in one step.) The size of the hedge is the amount of exposure to the real asset acquired in this way. The amount of hedge that is optimal will obviously depend on the properties of the fixed, preexisting portfolio.

We can think of the optimal hedge, expressed as a fraction of portfolio value, as having two components.⁵ The first is what we might call the *target* exposure — the level of exposure to a real asset that an investor ultimately wants to have. The second component is the exposure of the initial portfolio, or the *preexisting* exposure. The hedge is just the difference between the target of preexisting exposures.

Expressed in equations, this can be written:

$$\text{Optimal Hedge Ratio} =$$

$$\text{Target Exposure} - \text{Preexisting Exposure}$$

Standard mean-variance portfolio theory tells us that the target exposure depends on risk tolerance and the Sharpe ratio of an asset:

$$\text{Target Exposure} =$$

$$\text{Risk Tolerance} \times \text{Expected Excess Return of}$$

$$\text{Hedge Asset} \div \text{Variance of Hedge Asset}$$

where the excess return is the return in excess of the riskless rate.

Mean-variance analysis also tells us that preexisting exposure depends on the initial portfolio:

$$\text{Preexisting Exposure} = \text{Exposure of}$$

$$\text{Preexisting Portfolio to Hedge Asset}$$

Notice that target exposure depends *only* on the investor's risk tolerance and the risk/return ratio of the hedge asset. It has nothing to do with covariances of the hedge asset with other assets. On the other hand, the preexisting exposure has nothing to do with investor preferences or expected returns; it depends *only* on the covariance of the hedge with assets in the preexisting portfolio and the variance of hedge returns. Transaction costs are ignored here, but can easily be incorporated into the formulas (see Froot and Pérolé [1993]).

Such decomposition of the optimal hedge is useful because it separates out the speculative versus diversification motives for exposures to real assets. The risk-tolerant investor takes on hedging positions that are very sensitive to changes in expected returns, and relatively insensitive to preexisting exposures. The investor with little risk tolerance, on the other hand, generally has little desire to hold speculative exposures. Such an investor therefore hedges out the portfolio's preexisting exposure, to arrive at a near-zero target exposure. Since most real assets inherit inflation's negative correlation with broad portfolios, this hedge would imply diversifying — taking a long position to hedge portfolio risk.

DATA

We use three broad categories of real asset returns: inflation (in order to approximate the returns on a "real" inflation-linked bond), commodity price changes, and real estate returns. Exhibits 1 and 2 present summary statistics for these data during the entire 1970-1993 and 1983-1993 sample periods.

The specific data for inflation are:

EXHIBIT 1

Descriptive Statistics for Total Returns on Real Assets
(quarterly data, 1970-1993)

	Mean (%)	Variance (%) ^a	ρ_1^b	Number of Observations
<i>unxpi</i>	0.00	0.01	0.12	94
<i>infla</i>	5.75	0.03	0.89	94
<i>gsci</i>	4.86	2.06	0.11	94
<i>crb</i>	3.01	1.09	0.19	94
<i>gold</i>	10.09	5.39	0.22	94
<i>oil</i>	7.35	11.34	0.01	94
<i>cmeq</i>	8.71	1.75	0.22	94
<i>reit</i>	-8.88	8.47	0.24	88
<i>eval</i>	8.44	0.15	0.78	90
<i>cpihouse</i>	5.94	0.05	0.83	94
<i>adj</i>	7.47	1.17	0.60	85

^aAnnualized return variance.

^b ρ_1 indicates the first-order autocorrelation coefficient.

unxpi — unexpected inflation, measured as the unpredictable element of monthly percentage changes in the U.S. consumer price index using past CPI changes and U.S. Treasury bill rates; and

infla — U.S. inflation, measured as the percentage change in the CPI.

Alternative models of unexpected inflation produce very similar results, so are not reported here.

It should be noted that total inflation, *infla*, is perfectly correlated with the return on a CPI-linked bill only if the real interest rate is constant. This is a very restrictive assumption; we nevertheless include *infla* in the results below because the excess return on *infla* — inflation less the T-bill rate — is the most commonly used measure of unexpected inflation.

For commodity-related investments we use several measures (end-of-period prices):

gsci — percentage changes in the Goldman Sachs Commodity Index, a world production-weighted index of liquid exchange-traded commodity spot-market prices. This index is chosen because its weights approximate those of the PPI;⁶

crb — percentage changes in the Commodities Research Bureau Spot Index, an equally weighted average of the spot prices of traded commodities;

gold — percentage changes in the spot price of gold; and
oil — percentage changes in the spot price of crude oil (near-end-of-month prices from wellhead transactions in Brent crude).

The commodity prices are measured using spot (rather than rolled futures) prices. Investments in spot inventories are somewhat impractical, though, so most investors prefer to use futures markets to gain commodity exposure. Because futures markets were not functioning for several of these commodities over the entire sample period — most notably, oil futures were not very actively traded until 1983 — we use spot prices in much of the analysis instead.

It is worth noting that substituting spot for futures return data is not completely innocuous. Spot returns differ from those of collateralized futures. (To see the empirical importance of this difference, compare the total returns on spot and futures investments reported in Exhibit 2; also see Exhibit 11 below.)

Specifically, the return on the collateralized futures position is equal to the spot return plus two additional components: the short-term interest rate (from the financial collateral) plus the futures' roll yield. This sum is positive when futures prices fall below carry-adjusted spot prices. In such a case, "rolling" — i.e., following a strategy of buying the future, selling it

EXHIBIT 2

Descriptive Statistics for Total Returns on Real Assets
(quarterly data, 1983-1993)

	Mean (%)	Variance (%) ^a	ρ_1^b	Number of Observations
<i>unxpi</i>	0.00	0.00	-0.17	42
<i>infla</i>	3.76	0.01	0.80	42
<i>gsci</i> : spot	-2.92	2.93	-0.19	42
<i>gsci</i> : futures	12.09	3.35	-0.13	42
<i>crb</i> : spot	0.24	0.76	0.03	42
<i>crb</i> : futures	5.10	0.90	-0.29	28
<i>gold</i> : spot	-1.53	1.77	-0.14	42
<i>gold</i> : futures	-1.90	1.76	-0.07	42
<i>oil</i> : spot	-5.04	15.56	-0.20	42
<i>oil</i> : futures	9.85	22.12	-0.18	41
<i>cmeq</i>	14.00	1.36	0.20	42
<i>reit</i>	-8.29	3.38	0.11	36
<i>eval</i>	5.19	0.17	0.71	42
<i>cpihouse</i>	3.53	0.00	0.93	42
<i>adj</i>	0.68	1.04	0.55	42

^aAnnualized return variance.

^b ρ_1 indicates the first-order autocorrelation coefficient.

as it approaches maturity, and then purchasing a new longer-dated future — generates a positive “roll” yield.

For some commodities, such as gold, the sum of the interest rate and roll yield approximately net out, so that the return from buying and rolling (collateralized) gold futures is about equal to the return on holding inventories of gold. For other commodities, however — particularly oil — the interest rate plus roll yield has been considerably greater than zero.

Exhibit 3 presents data on the Treasury bill rate plus the roll yield for most major liquid commodities over the 1970–1993 period. It demonstrates that, historically, the return differential between collateralized futures and spot investments is positive and large: Across individual commodities, collateralized futures have outperformed spot investments by an average of 6.30% per year.

For crude oil, the corresponding number (calculated from 1983 to 1993) is 13.57%. Because the estimates below use spot returns, they ignore this return increment. Wherever this component may have an impact on the results, we provide commentary and additional estimates for the shorter period over which futures returns are available.

The last commodity-related investment alternative is to buy the equity of commodity-producing companies. To analyze the properties of this alternative we include the series:

cmeg — return on an index of primarily commodity-producing companies’ stocks. The index is composed of S&P sector-specific stock return indexes for: oil, aluminum, steel, miscellaneous metals, and foods. These sectors are then combined into a single index, using approximate relative GNP weights of the commodities they are intended to represent: 0.5, 0.03, 0.03, 0.03, and 0.4, respectively.

Measuring the returns on real estate investments is, of course, even more problematic than for commodities. First, real estate is a diverse class of assets, including commercial, residential, industrial, and retail investments, each driven in part by geographic factors.

Second, as is well known, most indexes of real estate returns are of poor quality, in the sense that they do not accurately measure actual holding-period returns. Real estate properties trade infrequently, so most indexes estimate value using appraisals or other estimates in lieu of transaction prices. Because appraisals

are conducted infrequently, and because even a current appraisal may not incorporate all recent market developments, appraisal-based indexes include predominantly “stale” prices. Staleness tends to make return data appear “smoothed” and of low volatility. One implication is that only a small component of each period’s index return is related to current changes in the properties’ market value — much of the “news” in the current index return is the predictable result of past changes in market conditions.⁷

Fortunately, there are several ways of eliminating or mitigating these data problems. One way is to use statistical procedures that intend to isolate the portion of appraisal-based returns that is related to contemporaneous changes in market prices, although there is no guarantee that these methods produce accurate measures of true holding-period returns, particularly in samples limited to post-war data (see Ankrum and Hensel [1992]).

A second way to mitigate these data problems is to avoid appraisal-based series altogether. One can, for example, assume the return on real estate properties is equal to the return on the stocks of companies that hold primarily real estate assets, such as REITs. The quality of these returns is much higher, as they are based on actual transaction prices.

Yet an investment in the stock of a REIT is not exactly the same as an outright purchase of real estate: REITs are managed by someone else, and often use a large amount of debt finance (on average over 50%). High levels of debt finance imply that, even if the value of the properties remains constant, the REIT stock price can change with changes in the value of the debt (due, say, to interest rate movements). Nevertheless, a large component of REIT returns will still reflect real estate itself.⁸

Our strategy is to use both original and statistically adjusted appraisal-based series, as well as an index of REIT returns. The specific series are:

reit — the S&P subindex of REIT stock returns.
eval — quarterly appraisal-based return index from Evaluation Associates, Inc. (a real estate analysis group located in Westport, Connecticut). These returns are on a diversified portfolio of open-ended real estate equity funds, and run from the second quarter of 1970 to the second quarter of 1993.⁹
house — monthly survey series of the value of residen-

EXHIBIT 3

Sum of Roll and T-Bill Yield

Date	Crude	Heating	Gasoline	Cattle	Hogs	Wheat	Corn	Soy	Coffee	Cocoa	Cotton	Sugar	Gold	Silver	Platinum	Copper
1970	N/A	N/A	N/A	8.50	26.84	8.4%	4.79	4.81	N/A	0.40	-4.04	9.43	N/A	-4.49	-0.73	9.33
1971	N/A	N/A	N/A	18.85	-9.67	10.28	1.12	2.03	N/A	1.59	-8.21	0.72	N/A	-3.54	0.95	-0.23
1972	N/A	N/A	N/A	5.11	1.97	13.32	-6.10	7.88	N/A	-2.34	33.95	-5.64	N/A	-2.30	-0.09	-3.78
1973	N/A	N/A	N/A	-6.96	14.58	26.69	27.60	67.69	-8.27	41.14	19.05	20.36	N/A	-0.33	-4.16	25.44
1974	N/A	N/A	N/A	-5.54	-10.90	11.83	11.95	7.00	3.12	53.83	16.63	65.09	N/A	-1.44	-3.25	18.09
1975	N/A	N/A	N/A	28.63	16.84	1.85	11.32	3.26	4.04	37.68	-4.69	18.99	-1.02	-3.69	-2.22	-5.68
1976	N/A	N/A	N/A	-9.52	39.30	-8.81	6.06	-2.36	15.25	32.84	2.27	-14.30	0.18	-0.52	-1.85	-3.51
1977	N/A	N/A	N/A	12.41	24.80	-10.74	-8.27	24.18	40.22	42.06	8.25	-18.38	0.53	-1.32	-1.17	-4.08
1978	N/A	N/A	N/A	2.48	17.68	4.84	-1.48	10.37	45.87	25.01	-5.96	-14.51	-1.22	-1.13	3.15	-3.59
1979	N/A	1.78	N/A	5.46	18.17	13.53	-1.49	3.82	20.77	-1.32	1.73	-14.44	-0.82	0.82	10.12	5.32
1980	N/A	-11.44	N/A	2.93	-9.67	-3.48	-5.46	-6.53	-3.09	5.32	15.30	-1.81	-3.52	-1.60	-0.39	2.80
1981	N/A	3.29	N/A	16.12	-3.67	-4.58	-1.16	0.60	20.32	-4.17	14.96	7.98	-0.54	-2.22	0.59	-2.28
1982	N/A	26.53	N/A	25.66	7.84	-9.21	-5.07	0.88	40.02	-7.13	-5.79	-15.57	-2.92	-1.44	1.91	-2.02
1983	N/A	11.94	N/A	14.00	18.18	-7.15	7.14	-2.36	15.15	0.87	2.79	-14.11	0.87	-1.16	3.44	-2.69
1984	10.29	21.75	N/A	13.93	-6.52	5.87	19.41	15.26	23.06	10.87	17.76	-30.81	-0.86	-1.04	0.41	-1.90
1985	30.77	19.05	37.40	0.61	0.75	10.92	8.81	1.57	7.30	10.67	8.21	-19.99	-0.07	-0.38	3.16	0.85
1986	9.85	18.81	18.10	23.09	28.08	40.17	16.62	5.56	0.30	-7.73	71.74	-20.44	0.42	-0.73	3.55	2.17
1987	17.95	5.83	2.81	20.30	44.35	6.89	-8.21	2.51	-5.75	-3.19	11.68	-18.21	-0.72	-1.44	1.20	13.66
1988	10.37	17.40	39.72	9.96	-3.90	-4.04	-7.71	-0.44	0.82	2.95	15.68	9.72	0.18	-1.58	4.80	49.29
1989	39.77	25.65	30.78	11.50	5.44	8.10	12.77	13.92	14.51	10.23	1.45	13.95	1.38	-1.36	6.39	-49.78
1990	-0.65	25.96	29.26	20.32	23.74	2.50	3.98	-4.07	-6.31	-3.16	21.03	8.88	0.43	-1.40	3.23	29.78
1991	18.55	16.68	28.37	3.32	19.11	-7.39	-5.37	-4.24	-10.98	-13.23	17.39	19.64	0.31	-1.35	1.26	12.19
1992	2.20	-5.87	-1.10	16.97	14.41	9.64	-8.38	-3.17	-14.67	-19.28	-6.65	14.23	0.19	-0.76	4.24	3.52
1993	-0.40	0.39	-8.35	13.75	10.93	19.44	-6.24	0.90	-13.61	-8.31	2.51	-9.07	-0.32	-0.22	2.46	-0.14
Average	13.57	11.85	19.67	10.50	12.15	5.79	2.78	6.21	8.96	8.57	10.29	0.03	-0.40	-1.44	1.55	3.87
Standard Deviation	12.96	11.71	16.70	9.97	14.87	11.91	9.79	14.55	17.51	19.22	16.68	20.60	1.18	1.14	3.12	17.07
T-Bill Average	6.32	7.99	5.93	7.25	7.25	7.25	7.25	7.25	7.57	7.25	7.25	7.25	7.50	7.25	7.25	7.25
Roll Yield Average	7.25	3.86	13.74	3.24	4.90	-1.46	-4.48	-1.04	1.39	1.32	3.04	-7.22	-7.96	-8.69	-5.71	-3.39

tial properties, conducted by the U.S. Bureau of Labor Statistics.

adj — an equally weighted average of the statistically adjusted versions of *eval* and *house*. Adjustments to remove the effects of “staleness” are made using a method similar to that in Ross and Zisler [1991]. To make these adjustments, we first specify an ad hoc model of the predictable component of index returns.

For both *eval* and *house* simple univariate autoregressive specifications are estimated:

$$\text{eval: } r_{i,t} = \alpha + 0.22r_{i,t-1} + 0.69r_{i,t-4} + \varepsilon_t; \\ (2.8) \quad (8.2)$$

$$R^2 = 0.63; DW = 1.97;$$

$$\text{house: } r_{i,t} = \alpha + 0.37r_{i,t-1} + 0.30r_{i,t-4} + \varepsilon_t; \\ (3.6) \quad (2.9)$$

$$R^2 = 0.34; DW = 1.88;$$

where $r_{i,t}$ is the time t return on index i , and t -statistics are in parentheses. The residual, ε_t , is proportional to the adjusted return, with a coefficient of proportionality derived using Ross and Zisler [1991]. This adjustment process increases the return variance of the unadjusted series by a factor of about 10.

MEASURING PREEXISTING EXPOSURES OF STANDARD PORTFOLIOS

Measuring the preexisting exposures of portfolios to real assets is straightforward. Exposure is equal to the coefficient in a regression of the portfolio excess return on the excess return of the real asset:

$$r_{p,t} = \alpha + \beta r_{ra,t} + \varepsilon_t$$

where $r_{p,t}$ is the time t excess return on some preexisting portfolio, β is the negative of the preexisting exposure of the portfolio, and $r_{ra,t}$ is the excess return on the real asset hedge.

To see this, suppose that for a given real asset, say, gold, we measure a regression β of -0.5 . That implies that a 1% up-move in the price of gold is on average associated with a 0.5% decline in the value of

the preexisting portfolio. If the value of the preexisting portfolio is, say, \$100 million, then this level of exposure implies that the portfolio has an exposure to gold of $-0.5 \times \$100 = -\50 million. To offset this exposure, one would need to go long the equivalent of \$50 million of gold.

The coefficient β is also equal to the minimum-variance hedge ratio, i.e., the hedge ratio that strips out the real assets' risks, and therefore minimizes total portfolio volatility. The amount by which the portfolio's risk falls is just the R^2 from the regression. We can also use the regression framework to test whether the exposure, β , is statistically significant.

It should be noted that standard portfolio theory provides no explicit means of incorporating uncertainty about exposures and correlations. If an exposure estimate is statistically insignificant, it is an indication only that the data cannot rule out a zero correlation, not a statement that a zero correlation is likely. Such statistical testing, however, is a way of asking the data to convince us that certain hedging properties do actually exist. Thus, a lack of statistical significance provides evidence consistent with those who argue that the true correlation is zero.

The top panel of Exhibit 4 reports estimates of the exposures of major asset classes to real asset returns on a quarterly basis from 1970 through 1993. (Annual return horizons were also estimated, and yield broadly similar results. Monthly return horizons yield slightly lower exposures, but similar qualitative results otherwise.)

The middle panel reports t -statistics of the coefficients. The bottom panel contains the associated R^2 values. Boldfacing indicates the coefficient estimates are statistically negative — i.e., that purchases of the real asset help reduce the volatility of the preexisting portfolio. Each column in the Exhibit corresponds to a particular real asset hedge, and each row corresponds to a particular preexisting portfolio.

The “preexisting portfolios” are: domestic stocks (measured by the returns on the S&P 500 from Ibbotson Associates); domestic bonds (measured by the Ibbotson series for total returns on medium-term U.S. government bonds); foreign stock returns (measured by the dollar return of the currency-unhedged EAFE index); foreign bond returns (measured as the average of dollar returns on medium-term currency-hedged government bonds in Japan, Germany, France, the U.K., Switzerland, and Canada); returns on currency hedges (measured as the dollar return of short-term currency

EXHIBIT 4

Exposures of Preexisting Portfolios to Real Asset Returns
(quarterly returns, 1973:1–1993:2)

Exposures Portfolio	Real Assets									
	<i>unxpi</i>	<i>infla</i>	<i>gsci</i>	<i>crb</i>	<i>gold</i>	<i>oil</i>	<i>cmeq</i>	<i>reit</i>	<i>eval</i>	<i>adj</i>
Domestic Stocks	-2.54	-1.98	-0.32	-0.24	-0.04	-0.14	1.09	0.35	-0.15	-0.27
Domestic Bonds	-2.52	-2.64	-0.30	-0.42	-0.03	-0.09	0.21	0.09	-0.25	-0.19
Foreign Stocks	-3.61	-1.99	-0.32	-0.08	0.14	-0.17	0.88	0.32	1.14	-0.02
Foreign Bonds	-1.41	-0.70	-0.11	-0.09	-0.02	-0.04	0.08	0.03	-0.21	-0.11
Currencies	-5.33	1.73	-0.09	0.08	0.21	-0.03	0.24	-0.05	-1.75	-0.30
Diversified Portfolio	-2.79	-2.07	-0.30	-0.24	0.01	-0.12	0.78	0.26	0.07	-0.22
T-Statistics										
Portfolio	Real Assets									
	<i>unxpi</i>	<i>infla</i>	<i>gsci</i>	<i>crb</i>	<i>gold</i>	<i>oil</i>	<i>cmeq</i>	<i>reit</i>	<i>eval</i>	<i>adj</i>
Domestic Stocks	-2.68	-1.84	-2.63	-1.44	-0.51	-2.65	15.09	6.37	-0.28	-1.60
Domestic Bonds	-3.88	-3.63	-3.63	-3.84	-0.59	-2.35	2.27	1.92	-0.64	-1.47
Foreign Stocks	-3.65	-1.74	-2.49	-0.44	1.73	-3.26	8.12	5.56	2.05	-0.08
Foreign Bonds	-7.06	-2.66	-3.71	-2.12	-0.88	-2.83	2.37	1.95	-1.55	-2.38
Currencies	-5.78	1.46	-0.67	0.43	2.53	-0.49	1.67	-0.70	-3.08	-1.54
Diversified Portfolio	-3.92	-2.54	-3.32	-1.91	0.10	-3.22	12.68	6.25	0.16	-1.63
R ² s										
Portfolio	Real Assets									
	<i>unxpi</i>	<i>infla</i>	<i>gsci</i>	<i>crb</i>	<i>gold</i>	<i>oil</i>	<i>cmeq</i>	<i>reit</i>	<i>eval</i>	<i>adj</i>
Domestic Stocks	0.07	0.04	0.07	0.02	0.00	0.07	0.71	0.32	0.00	0.03
Domestic Bonds	0.14	0.13	0.13	0.14	0.00	0.06	0.05	0.04	0.00	0.03
Foreign Stocks	0.13	0.03	0.06	0.00	0.03	0.11	0.42	0.26	0.05	0.00
Foreign Bonds	0.36	0.07	0.13	0.05	0.01	0.08	0.06	0.04	0.03	0.07
Currencies	0.27	0.02	0.00	0.00	0.07	0.00	0.03	0.01	0.10	0.03
Diversified Portfolio	0.15	0.07	0.11	0.04	0.00	0.10	0.64	0.31	0.00	0.03

Boldface numbers represent statistically significant negative exposures at the 5% level.

forward contracts on the Canadian dollar, French franc, deutschemark, Japanese yen, and British pound); the returns on real estate portfolios (measured by the three real estate series mentioned above: *reit*, *eval*, and *adj*); and the returns on a broadly diversified portfolio (a 70/30 portfolio of stocks and government bonds, and a domestic/foreign mix of 75/25 in both stocks and bonds).

There are a number of noteworthy points to emerge from Exhibit 4. First, long positions in most of the real assets help hedge the diversified portfolios. The last line of the Exhibit shows a negative exposure of the diversified portfolio to most real assets. (A negative exposure to a real asset implies that a long position is needed to offset that exposure. In this sense, long positions in real assets are "hedges.")

The exceptions are two of the real estate series,

reit and *eval*, and the commodity-based equities series, *cmeq*. Indeed, the portfolio exposures to *reit* and *cmeq* are highly statistically *positive* in both the quarterly and annual return data, suggesting that long positions in these assets actually *add* to portfolio risk, rather than reduce it. In fact, most equity and bond investments show positive exposures that are similar to those of *reit* and *cmeq*.

In this sense, these investments — which are both traded on stock exchanges — appear to trade more like stocks than they do like real assets. (The resemblance of REIT returns to those of stocks has been noted by Gyourko and Keim [1992] among others.)

It is not clear how to interpret these positive exposure estimates, particularly the estimate for the exchange-traded real estate series, *reit*. Some observers

have argued that returns of exchange-traded real estate are “excessively” correlated with other stocks, the result of noise traders who drive all stock exchange prices temporarily up and down relative to true value. The underlying true values, the argument goes, are much less positively (and probably negatively) correlated.

A second explanation of the positive correlations is that *true* underlying stock market and real estate values are, in fact, highly positively related in the first place. The reasoning here is that very similar factors — e.g., domestic productivity of capital and labor — ought to drive both stock prices and real estate values, and that, in any case, approximately 25% of the corporate assets are real estate in the first place. This alternative argument supports the view that REIT values are reasonably accurate measures of underlying real estate values.

This brings us to the second point to emerge from Exhibit 4: There is only weak evidence at best that real estate is a useful hedging tool. REITs do not appear to be candidates for hedges, and both the unadjusted and adjusted appraisal-based series, *eval* and *adj*, show statistically insignificant hedging properties in diversified portfolios.

It should be noted that other authors have, in contrast, concluded that real estate can help hedge broad portfolios — particularly those of domestic bonds.¹⁰ Like these other studies, our data also provide evidence that real estate returns are positively correlated with both inflation and unexpected inflation. (For example, the exposure of *adj* to inflation is the highest among all three real estate series, at a statistically significant 2.08; the *reit* series is, like other stocks, negatively exposed to inflation.)

Nevertheless, in our data the exposures of stocks to *eval* and *adj* are not sufficiently negative to provide much evidence of hedging potential in diversified portfolios. Indeed, when these near-zero exposures are combined with the evidence from the *reit* data, it appears that real estate is at best marginally helpful in hedging diversified portfolios, even though real estate itself may be adequately hedged against inflation.

The third point to take from Exhibit 4 is that the most effective hedges (in terms of statistically significant negative exposures and high R^2 values) are unexpected inflation, GSCI returns, and oil price changes. Gold and the CRB index are not themselves good hedges — both are negatively correlated with the returns on diversified portfolios, but have low and statistically

insignificant R^2 values.

The negative exposures of diversified portfolios to *unxpi*, *gsci*, and *oil* are fairly evenly distributed across stocks and bonds, both domestic and foreign (see Exhibit 4, columns 1, 3, and 9, and rows 1 through 4). Note that the exposure of the diversified portfolio to unexpected inflation is particularly large in magnitude (at approximately -2.8 ; see line 6, column 1 in the top panel). This implies that the minimum-variance hedge ratio requires a long CPI-linked bond position equal to almost three times the value of the preexisting portfolio. The exposures of the broad portfolio to the GSCI and oil are much more modest, at about -0.30 and -0.12 , respectively.

Exhibit 5 reports an analogous set of estimates, but with only the most recent ten years of data. Besides singling out more recent data, the late starting date permits the use of futures commodity data. This comparison gives us a sense of the reliability of the spot commodity correlations in the longer time series samples above.

The estimates show that there is, in fact, little difference in the hedging properties of spot versus futures positions in commodities. GSCI and oil futures appear to offer hedging properties that are as effective as those of comparable spot investments.¹¹

Perhaps the most striking result in Exhibit 5 is the effectiveness of the GSCI and oil hedges in comparison with other, non-commodity hedging vehicles. In particular, it appears that unexpected inflation has become a much less effective hedge during the recent period — the magnitude and statistical significance of portfolio exposures to both *unxpi* and *infla* have declined substantially.

At the same time, the R^2 s of the GSCI and oil appear to have increased during the recent period. This suggests that during periods of relatively low inflation, broad portfolios are less exposed to inflation risk, but remain highly exposed to commodity price risks.

The same finding — that commodity prices can be better hedges than unexpected inflation — also emerges from much longer time series data. Using annual returns from 1947 through 1992, Exhibit 6 shows that the preexisting exposure of a U.S. domestic stock/bond portfolio to the GSCI is -0.35 , with an R^2 of 0.19. By comparison, the exposure to unexpected inflation is -0.77 (with an R^2 of only 0.09), and the exposure to inflation itself is -1.14 (with an R^2 of 0.10).

Like the previous two exhibits, the results of

Exhibit 6 suggest that almost any combination of commodities does at least reasonably well in protecting bond portfolios against inflation. Oil, however (with or without other energy prices), is needed to hedge stock portfolios effectively.

Is it possible that commodities futures positions can reduce as much or even more risk than unexpected inflation hedges? The answer is yes, especially if stocks respond negatively to commodity price movements when inflation remains unchanged. That is, if the *relative* price of industrial inputs to outputs is more important to businesses than overall inflation, then commodity inflation will be more effective than CPI inflation for hedging stocks. Indeed, it is even plausible that the well-known negative correlation between

stock prices and inflation is actually driven by changes in relative input prices, which, after all, are highly correlated with inflation.¹²

An alternative explanation for these results is that they understate actual exposures to unexpected inflation. This can occur because unexpected inflation is unobservable, so that *unxpi* is likely to measure it with error.

At the same time, there is evidence that measurement error is not a serious problem. First, we estimated a number of different models for unexpected inflation, all of which yielded results similar to those reported. Second, actively traded CPI-linked bonds exist in several countries, such as Canada and the U.K. Wadhvani and Shah [1993] find that the GSCI index appears to be a better hedge for U.K. portfolios than are

EXHIBIT 5

Exposures of Preexisting Portfolios to Real Asset Returns

(quarterly returns, 1983:1-1993:2, using rolled futures for commodity returns)

Exposures Portfolio	Real Assets									
	<i>unxpi</i>	<i>infla</i>	<i>gsci</i>	<i>crb</i>	<i>gold</i>	<i>oil</i>	<i>cmeq</i>	<i>reit</i>	<i>eval</i>	<i>adj</i>
Domestic Stocks	-0.89	-1.25	-0.25	-0.34	-0.34	-0.11	1.00	0.26	-0.37	-0.24
Domestic Bonds	-2.56	-2.81	-0.25	-0.23	-0.07	-0.11	0.03	0.02	-0.38	-0.11
Foreign Stocks	-2.70	-4.39	0.07	0.18	0.20	-0.08	0.95	0.38	1.32	0.12
Foreign Bonds	-1.77	-0.80	-0.09	-0.15	-0.01	-0.04	0.05	0.00	-0.27	-0.07
Currencies	-7.57	3.53	0.06	-0.23	0.65	0.01	-0.20	-0.40	-2.37	-0.52
Diversified Portfolio	-1.86	-2.46	-0.21	-0.23	-0.15	-0.10	0.70	0.19	-0.09	-0.21

T-Statistics

Portfolio	Real Assets									
	<i>unxpi</i>	<i>infla</i>	<i>gsci</i>	<i>crb</i>	<i>gold</i>	<i>oil</i>	<i>cmeq</i>	<i>reit</i>	<i>eval</i>	<i>adj</i>
Domestic Stocks	-0.49	-0.64	-2.01	-1.06	-2.03	-2.21	7.44	1.80	-0.57	-1.04
Domestic Bonds	-2.07	-2.08	-2.89	-1.31	-0.59	-3.23	0.23	0.23	-0.81	-0.67
Foreign Stocks	-1.25	-1.91	-0.44	0.49	0.96	-1.39	5.10	2.56	1.84	0.40
Foreign Bonds	-3.88	-1.39	-2.43	-2.04	-0.23	-2.32	0.82	0.08	-1.48	-1.03
Currencies	-4.09	1.51	0.41	-0.62	3.41	0.18	-0.79	-2.68	-3.38	-1.90
Diversified Portfolio	-1.34	-1.64	-2.21	-1.02	-1.09	-2.72	6.65	1.83	-0.19	-1.10

R²s

Portfolio	Real Assets									
	<i>unxpi</i>	<i>infla</i>	<i>gsci</i>	<i>crb</i>	<i>gold</i>	<i>oil</i>	<i>cmeq</i>	<i>reit</i>	<i>eval</i>	<i>adj</i>
Domestic Stocks	0.01	0.01	0.09	0.04	0.09	0.11	0.58	0.09	0.01	0.03
Domestic Bonds	0.10	0.10	0.17	0.06	0.01	0.21	0.00	0.00	0.02	0.01
Foreign Stocks	0.04	0.09	0.01	0.01	0.02	0.05	0.41	0.16	0.08	0.00
Foreign Bonds	0.28	0.05	0.13	0.10	0.00	0.12	0.02	0.00	0.05	0.03
Currencies	0.29	0.05	0.00	0.01	0.22	0.00	0.02	0.17	0.22	0.08
Diversified Portfolio	0.05	0.07	0.11	0.04	0.03	0.17	0.54	0.09	0.00	0.03

Boldface numbers represent statistically significant negative exposures at the 5% level.

EXHIBIT 6

Exposures of Preexisting Portfolios to Real Asset Returns
(annual returns, 1947-1992)

Exposures Portfolio	Real Assets				
	<i>unxpi</i>	<i>infla</i>	<i>gsci</i>	<i>crb</i>	<i>gold</i>
Domestic Stocks	-1.11	-1.41	-0.38	-0.15	-0.13
Domestic Bonds	0.02	-0.49	-0.28	-0.20	-0.11
Diversified Portfolio*	-0.77	-1.14	-0.35	-0.16	-0.12

T-Statistics Portfolio	Real Assets				
	<i>unxpi</i>	<i>infla</i>	<i>gsci</i>	<i>crb</i>	<i>gold</i>
Domestic Stocks	-2.30	-2.11	-2.51	-0.87	-1.12
Domestic Bonds	0.08	-1.28	-3.51	-2.22	-1.90
Diversified Portfolio*	-2.10	-2.27	-3.18	-1.29	-1.47

R ² s Portfolio	Real Assets				
	<i>unxpi</i>	<i>infla</i>	<i>gsci</i>	<i>crb</i>	<i>gold</i>
Domestic Stocks	0.11	0.09	0.12	0.02	0.03
Domestic Bonds	0.06	0.04	0.22	0.10	0.08
Diversified Portfolio*	0.09	0.10	0.19	0.04	0.05

*This is a 70/30 weighted portfolio of domestic stocks (S&P 500) and domestic bonds (U.S. government bonds).

Boldface numbers represent statistically significant negative exposures at the 5% level.

British long-maturity CPI-linked bonds.

Comparing and Combining Real Asset Hedges

It would be rare for a portfolio manager to rely on a single real asset to hedge a portfolio of bonds and stocks. If real asset returns were uncorrelated with one another, then exposures to each could be adjusted separately, and one could rely on the regressions for guidance on the efficacy of each hedge. Most real asset returns are positively correlated, however, creating more of an either/or decision when choosing hedges. We need to examine how real hedges interact with one another, in order to understand better how they might be combined.

The method we use is just an extension of the exposure measurement above. Now we want to measure the exposures of a preexisting portfolio to combinations of real asset hedges. To do this, we first implement the minimum-variance hedge for each of the real assets

individually. Then we ask whether *other* real assets can provide evidence of further hedging the portfolio.

This involves regressing the diversified portfolio's return, $r_{p,t}$, on one of the real asset returns, $r_{ra1,t}$, taking the residuals, ϵ_{t1} , and then regressing them on other real asset returns, $r_{ra2,t}$:

$$\text{Initial Hedge Regression: } r_{p,t} = \alpha_1 + \beta_1 r_{ra1,t} + \epsilon_{t1};$$

Secondary Hedge Regression:

$$\epsilon_{t1} = \alpha_{1,2} + \beta_{1,2} r_{ra2,t} + \epsilon_{t2}$$

The resulting secondary exposure, $\beta_{1,2}$, applies to the preexisting portfolio inclusive of the initial hedge's returns. The statistical significance of $\beta_{1,2}$ reveals whether the secondary minimum-variance hedge reduces portfolio variance.¹³ The coefficient of determination of the secondary regression, ρ^2 , indicates the amount of portfolio variance reduction offered by the secondary hedge.

In this setup, the most potent hedges will exhibit two properties. First, they should still be useful for reducing portfolio volatility even after a minimum-variance hedge in another real asset has been executed; and second, once a minimum-variance hedge in a potent hedge has been executed, other additional hedges should have little additional impact on return volatility. Weaker hedging vehicles will lack one or both of these properties.

Exhibit 7 allows us to assess these properties in the group of real assets over 1970-1993. To conserve on space, we report the secondary exposures for only one preexisting portfolio — the stock/bond diversified portfolio. In addition, we report the t-statistics testing the likelihood that the secondary exposures are equal to zero, plus the overall R²s of the regression.

The results show that unexpected inflation, the GSCI, and crude oil all basically satisfy both of the properties above. They basically retain their stand-alone exposures (and their statistical significance as volatility-reducing hedges) when they are combined with other real assets, such as gold, the CRB index, or real estate.

For example, when the GSCI is the initial minimum-variance hedge, it renders the other hedge assets ineffective (except for *cmeq* and *reit*, which are positively correlated with the diversified portfolio). To see this, note that the GSCI row of Exhibit 7's t-statistic panel shows that few of the secondary hedges are sta-

EXHIBIT 7

Co-Exposures of Preexisting Portfolios to Real Asset Returns (quarterly returns, 1970-1993)

Initial Hedge Regressions of $r_{p,t} = \alpha_1 + \beta_1 r_{ra1,t} + \varepsilon_{t1}$;

Secondary Hedge Regressions of $\varepsilon_{t1} = \alpha_2 + \beta_{1,2} r_{ra2,t} + \varepsilon_{t2}$

Exposures to Secondary Hedge $\beta_{1,2}$

Initial Hedge	<i>unxpi</i>	<i>infla</i>	<i>gsci</i>	<i>crb</i>	Secondary Hedge		<i>cmeq</i>	<i>reit</i>	<i>eval</i>	<i>adj</i>
					<i>gold</i>	<i>oil</i>				
<i>unxpi</i>		-1.57	-0.22	0.31	-0.01	-0.09	0.72	0.23	0.38	-0.12
<i>infla</i>	-2.48		-0.17	-0.14	0.06	-0.09	0.77	0.23	0.06	-0.12
<i>gsci</i>	-2.25	-0.64		-0.04	0.06	-0.02	0.77	0.23	0.23	-0.12
<i>crb</i>	-3.00	-1.59	-0.21		0.04	-0.11	0.78	0.26	0.18	-0.17
<i>gold</i>	-2.78	-2.10	-0.31	-0.24		-0.13	0.78	0.26	0.06	-0.23
<i>oil</i>	-2.29	-1.17	-0.06	-0.18	0.03		0.77	0.24	0.20	-0.12
<i>cmeq</i>	-1.76	-1.78	-0.28	-0.21	-0.02	-0.11		0.05	0.07	-0.15
<i>reit</i>	-2.34	-0.92	-0.20	-0.23	0.01	-0.09	0.47		-0.53	-0.32
<i>eval</i>	-2.79	-2.00	-0.31	-0.25	0.00	-0.13	0.77	0.24		-0.24
<i>adj</i>	-2.53	-1.59	-0.29	-0.22	0.02	-0.11	0.76	0.26	0.45	

T-Statistics of Secondary Hedge

Initial Hedge	<i>unxpi</i>	<i>infla</i>	<i>gsci</i>	<i>crb</i>	Secondary Hedge		<i>cmeq</i>	<i>reit</i>	<i>eval</i>	<i>adj</i>
					<i>gold</i>	<i>oil</i>				
<i>unxpi</i>		-2.07	-2.61	-2.68	-0.11	-2.56	12.38	5.97	1.04	-0.97
<i>infla</i>	-3.57		-1.90	-1.16	1.12	-2.34	13.43	5.51	0.15	-0.88
<i>gsci</i>	-3.28	-0.81		-0.37	1.12	-0.56	14.54	5.69	0.59	-0.91
<i>crb</i>	-4.37	-1.96	-2.34		0.71	-2.95	13.14	6.39	0.46	-1.24
<i>gold</i>	-3.91	-2.59	-3.35	-1.94		-3.24	12.65	6.26	0.15	-1.65
<i>oil</i>	-3.34	-1.48	-0.70	-1.49	0.53		14.01	5.95	0.53	-0.92
<i>cmeq</i>	-4.18	-3.80	-5.62	-2.86	-0.67	-5.02		1.68	0.29	-1.83
<i>reit</i>	-3.71	-1.31	-2.53	-2.16	-0.13	-2.79	6.66		-1.43	-2.77
<i>eval</i>	-4.02	-2.50	-3.51	-2.00	0.04	-3.40	11.88	5.65		-1.74
<i>adj</i>	-3.42	-1.95	-3.15	-1.76	0.27	-2.89	11.79	6.06	1.08	

R²s of Secondary Hedge

Initial Hedge	<i>unxpi</i>	<i>infla</i>	<i>gsci</i>	<i>crb</i>	Secondary Hedge		<i>cmeq</i>	<i>reit</i>	<i>eval</i>	<i>adj</i>
					<i>gold</i>	<i>oil</i>				
<i>unxpi</i>		0.05	0.07	0.07	0.00	0.07	0.63	0.29	0.01	0.01
<i>infla</i>	0.12		0.04	0.01	0.01	0.06	0.67	0.26	0.00	0.01
<i>gsci</i>	0.11	0.01		0.00	0.01	0.00	0.70	0.27	0.00	0.01
<i>crb</i>	0.18	0.04	0.06		0.01	0.09	0.66	0.32	0.00	0.02
<i>gold</i>	0.15	0.07	0.11	0.04		0.10	0.64	0.31	0.00	0.03
<i>oil</i>	0.11	0.02	0.01	0.02	0.00		0.69	0.29	0.00	0.01
<i>cmeq</i>	0.16	0.14	0.26	0.08	0.00	0.22		0.03	0.00	0.04
<i>reit</i>	0.14	0.02	0.07	0.05	0.00	0.08	0.34		0.02	0.09
<i>eval</i>	0.16	0.07	0.13	0.04	0.00	0.12	0.62	0.28		0.04
<i>adj</i>	0.13	0.04	0.11	0.04	0.00	0.09	0.63	0.32	0.01	

Boldface numbers represent statistically significant negative exposures at the 5% level.

tistically useful in reducing what remains of portfolio variance after the GSCI minimum-variance hedge has been implemented. On the other hand, when the GSCI is the follow-on or secondary hedge, it remains statistically significant (see the GSCI column of the t-

statistics panel).

The same cannot be said for real estate. Exhibit 7 shows that once the real estate hedges have been implemented, exposures to other real assets (such as unexpected inflation, the GSCI, and oil) remain just as

large. On the other hand, real estate does little to reduce portfolio volatility once most other hedges have already been put in place.

Exhibit 8 reports analogous results for the most recent decade only, again showing futures data for the commodities. (The results for spot data over the same sample period are very similar, and therefore are omitted.) Here, unexpected inflation loses much of its negative correlation with the stock/bond portfolio, so that the most potent hedging combination is that of the GSCI and oil, which together can reduce as much as about 25% of preexisting portfolio variance.

The results are similar for the longer time series sample, estimates of which are shown in Exhibit 9. The main difference in Exhibit 9 is that the time sample is large enough to give considerable statistical significance to the GSCI exposure estimates in all cases.

Summary

To sum up so far, the evidence above suggests that broad portfolios have large negative preexisting exposures to some commodities (such as the GSCI and oil) and to unexpected inflation. We find little evidence of a strong negative exposure to real estate or to commodity-based equities. Data problems notwithstanding, this raises questions about the usefulness of real estate for hedging diversified stock and bond portfolios.

Furthermore, there is even less evidence that real estate or commodity-based equity investments can help reduce portfolio variance once unexpected inflation, the GSCI, or oil are included in the preexisting portfolio. In effect, these latter assets appear to strip out most of the portfolio risks that one looks to real assets to protect against.

Finally, there is some evidence that even if the perfect CPI-inflation hedge were available in the U.S. (i.e., U.S. CPI-linked government bills and bonds), its use would not crowd out a hedging role for commodities such as the GSCI and oil.

TARGET EXPOSURES

Optimal target exposure is more difficult to determine than preexisting exposure. That is because target exposures depend on investor risk tolerance and future expected returns, both of which are unobservable. Nevertheless, we can discuss what different assumptions about these quantities have to say about optimal target exposures.

Investor Risk Tolerance

Investor risk tolerance reflects the investor's trade-off between expected return and risk (expressed in the framework we have been using as the variance of excess returns). In order to give a rough sense of scale, we calibrate the figures so that a risk tolerance of 0.5 applies to an investor who is willing to accept an increased portfolio variance of 1% in order to increase expected return by 100 basis points.

Using the average returns and volatility of stock and bond portfolios over the last twenty years, Exhibit 10 gives the implied risk tolerance of an investor whose preexisting portfolio consists entirely of: domestic stocks; domestic bonds; foreign stocks; foreign bonds; and a 70/30 stock/bond diversified portfolio with a 75/25 domestic/foreign mix.

Exhibit 10 shows that an investor who holds only U.S. domestic stocks would have an implied risk tolerance of 0.51. Alternatively, an investor who holds the diversified portfolio has a risk tolerance of 0.35. Assuming that today's excess expected return on the diversified portfolio is 3.0%, the implied risk tolerance of an investor who holds that portfolio rises to 0.43. Alternatively, if one expects a future excess return of 5.0%, implied risk tolerance drops to 0.26.

Given these rough computations, a generous range for risk tolerance might be between 0.25 and 0.50.

Expected Excess Returns

Expected returns are also difficult to gauge. While historical data tend to provide relatively accurate estimates of future portfolio variance, the same data provide only imprecise estimates of future expected returns. Exhibit 11 reports the historical average excess returns.

The imprecision of estimated expected returns is a problem for all these real assets. For the inflation measures, one must recall that unexpected inflation is not a traded security. If CPI-linked bills were in fact traded, their expected excess return over T-bills would probably be negative.

Expected returns on commodities and real estate are also problematic. First, for commodities — particularly oil — low frequency price movements make it risky to extrapolate out past average returns. For example, it is likely that the average return on oil over 1970-1993, which includes little net change in OPEC's market power, is more representative of future returns than the return from the past decade.

EXHIBIT 8

Co-Exposures of Preexisting Portfolios to Real Asset Returns
(quarterly returns, 1983-1993)

Initial Hedge Regressions of $r_{p,t} = \alpha_1 + \beta_1 r_{ra1,t} + \varepsilon_{t1}$;

Secondary Hedge Regressions of $\varepsilon_{t1} = \alpha_2 + \beta_{1,2} r_{ra2,t} + \varepsilon_{t2}$

Exposures to Secondary Hedge $\beta_{1,2}$

Initial Hedge	<i>unxpi</i>	<i>infla</i>	<i>gsci</i>	<i>crb</i>	Secondary Hedge		<i>cmeq</i>	<i>reit</i>	<i>eval</i>	<i>adj</i>
					<i>gold</i>	<i>oil</i>				
<i>unxpi</i>		-2.44	-0.17	-0.23	-0.18	-0.08	0.73	0.18	0.17	-0.16
<i>infla</i>	-1.84		-0.15	-0.20	-0.11	-0.07	0.71	0.17	-0.27	-0.19
<i>gsci</i>	-1.00	-1.35		-0.08	-0.02	-0.03	0.76	0.18	0.07	-0.14
<i>crb</i>	-1.53	-2.27	-0.12		-0.19	-0.07	0.65	0.13	-0.30	-0.19
<i>gold</i>	-2.11	-2.16	-0.16	-0.17		-0.08	0.71	0.16	-0.13	-0.24
<i>oil</i>	-0.59	-0.87	0.00	-0.16	-0.01		0.75	0.16	-0.01	-0.15
<i>cmeq</i>	-2.58	-2.64	-0.28	-0.23	-0.16	-0.12		-0.01	-0.39	-0.23
<i>reit</i>	-3.01	-1.81	-0.21	-0.27	-0.10	-0.09	0.57		-0.61	-0.36
<i>eval</i>	-1.76	-2.53	-0.20	-0.22	-0.15	-0.10	0.71	0.20		-0.18
<i>adj</i>	-1.56	-2.35	-0.19	-0.18	-0.17	-0.09	0.71	0.23	0.29	

T-Statistics of Secondary Hedge

Initial Hedge	<i>unxpi</i>	<i>infla</i>	<i>gsci</i>	<i>crb</i>	Secondary Hedge		<i>cmeq</i>	<i>reit</i>	<i>eval</i>	<i>adj</i>
					<i>gold</i>	<i>oil</i>				
<i>unxpi</i>		-1.66	-1.82	-1.02	-1.35	-2.21	7.56	1.88	0.36	-0.85
<i>infla</i>	-1.37		-1.65	-0.89	-0.83	-2.06	7.34	1.64	-0.58	-1.03
<i>gsci</i>	-0.76	-0.93		-0.36	-0.16	-0.79	9.52	1.901	0.16	-0.76
<i>crb</i>	-0.89	-0.98	-1.21		-1.25	-1.40	6.79	1.04	-0.55	-0.93
<i>gold</i>	-1.56	-1.45	-1.70	-0.76		-2.25	6.93	1.58	-0.27	-1.28
<i>oil</i>	-0.45	-0.59	0.03	-0.75	-0.05		10.02	1.67	-0.02	-0.85
<i>cmeq</i>	-2.96	-2.73	-5.68	-1.71	-1.73	-6.41		-0.12	-1.19	-1.87
<i>reit</i>	-1.91	-1.13	-2.26	-1.13	-0.74	-2.62	4.65		-1.21	-1.88
<i>eval</i>	-1.26	-1.69	-2.18	-0.99	-1.11	-2.70	6.74	1.93		-0.96
<i>adj</i>	-1.13	-1.59	-2.02	-0.81	-1.27	-2.61	6.99	2.33	0.60	

R²s of Secondary Hedge

Initial Hedge	<i>unxpi</i>	<i>infla</i>	<i>gsci</i>	<i>crb</i>	Secondary Hedge		<i>cmeq</i>	<i>reit</i>	<i>eval</i>	<i>adj</i>
					<i>gold</i>	<i>oil</i>				
<i>unxpi</i>		0.07	0.08	0.04	0.05	0.12	0.60	0.09	0.00	0.02
<i>infla</i>	0.05		0.07	0.03	0.02	0.10	0.59	0.07	0.01	0.03
<i>gsci</i>	0.01	0.02		0.01	0.00	0.02	0.70	0.10	0.00	0.01
<i>crb</i>	0.03	0.04	0.06		0.06	0.08	0.66	0.05	0.01	0.03
<i>gold</i>	0.06	0.05	0.07	0.02		0.12	0.56	0.07	0.00	0.04
<i>oil</i>	0.01	0.01	0.00	0.02	0.00		0.73	0.08	0.00	0.02
<i>cmeq</i>	0.19	0.16	0.46	0.11	0.07	0.53		0.00	0.04	0.08
<i>reit</i>	0.10	0.04	0.13	0.06	0.02	0.17	0.39		0.04	0.09
<i>eval</i>	0.04	0.07	0.11	0.04	0.03	0.17	0.54	0.10		0.02
<i>adj</i>	0.03	0.06	0.10	0.03	0.04	0.16	0.56	0.14	0.01	

Boldface numbers represent statistically significant negative exposures at the 5% level.

EXHIBIT 9

Co-Exposures of Preexisting Portfolios to Real Asset Returns
(annual returns, 1947-1992)

Initial Hedge Regressions of $r_{p,t} = \alpha_1 + \beta_1 r_{ra1,t} + \epsilon_{t1}$

Secondary Hedge Regressions of $\epsilon_{t1} = \alpha_2 + \beta_{1,2} r_{ra2,t} + \epsilon_{t2}$

Exposures to Secondary Hedge $\beta_{1,2}$		Secondary Hedge			
Initial Hedge		$gsci$	crb	$gold$	
	$unxpi$	$infla$			
$unxpi$		-0.28	-0.23	-0.06	-0.06
$infla$	-0.11		-0.18	-0.09	-0.04
$gsci$	-0.22	-0.15		0.03	0.03
crb	-0.62	-0.94	-0.27		-0.07
$gold$	-0.57	-0.79	-0.24	-0.07	

T-Statistics of Secondary Hedge

Initial Hedge		$gsci$	crb	$gold$	
	$unxpi$	$infla$			
$unxpi$		-0.56	-2.04	-0.53	-0.74
$infla$	-0.29		-1.59	-0.70	-0.47
$gsci$	-0.63	-0.31		0.23	0.36
crb	-1.64	-1.83	-2.33		-0.83
$gold$	-1.55	-1.56	-2.13	-0.58	

R²s of Secondary Hedge

Initial Hedge		$gsci$	crb	$gold$	
	$unxpi$	$infla$			
$unxpi$		0.01	0.09	0.01	0.01
$infla$	0.00		0.05	0.01	0.00
$gsci$	0.01	0.00		0.00	0.00
crb	0.06	0.07	0.11		0.02
$gold$	0.05	0.05	0.09	0.01	

Boldface numbers represent statistically significant negative exposures at the 5% level.

Second, as discussed above, spot and futures positions in commodities have historically resulted in very different excess returns. One might assume that the past excess returns on commodity futures will be sustained in the future, although improved commodity market efficiency is unlikely to raise roll yields, and very likely to lower them. Thus, the statistical imprecision of our estimates is amplified in the case of commodities, where it is not clear which benchmark — past spot or futures returns — is more relevant.

There is a similar problem with expected returns on real estate. For example, the *eval* and *adj* series total return indexes show declines of only about 5% and 14%, respectively, since their peaks in the late 1980s. It is unclear whether this decline reflects the full downturn in commercial real estate markets. Here, the unadjusted data appear smooth and unvolatile, but the degree to which average returns can be estimated is nevertheless imprecise. Furthermore, transaction costs (including costs of illiquidity) should also factor into the expected return on real estate, even though such considerations are not reflected in Exhibit 11.

Calculating Target Exposures and Optimal Hedge Ratios

We have looked at all the inputs into the target exposure and optimal hedge ratio decisions. We now combine them using the framework laid out above to gain a sense for the magnitudes and sensitivity of optimal real asset hedge ratios.

As indicated above, the calculation of hedge ratios requires three main inputs: investor risk tolerance; expected future excess returns; and preexisting portfolio exposures. In the calculations presented below, we assume investor risk tolerance is 0.5. We also discuss the sensitivity of the results to different assumptions about risk tolerance.

For expected returns, we try several different

EXHIBIT 10

Investor Risk Tolerance Implied by Historical Returns

	Domestic Stocks	Domestic Bonds	Foreign Stocks	Foreign Bonds	Diversified Portfolio
Historical Return	5.10%	2.40%	1.70%	3.00%	3.70%
Historical Volatility	16.10	11.20	17.00	9.80	11.40
Implied Risk Tolerance*	0.51	0.52	1.74	0.32	0.35

*Implied risk tolerance for each portfolio is calculated as the historical return variance divided by the historical average return.

EXHIBIT 11

Historical Excess Returns on Real Assets

(returns above the U.S. Treasury bill rate, in percent per year)

Real Asset	Historical Excess Returns: 1970-1993	Historical Excess Returns: 1983-1993
<i>unxpi</i>	0.0 ^a	0.0
<i>infla</i>	-1.3	-2.8
<i>gsci</i> : spot	-2.1	-9.5
futures	6.3 ^b	5.5
<i>crb</i> : spot	-4.0	-6.3
futures	1.0 ^c	0.6 ^c
<i>gold</i> : spot	3.1	-8.1
futures	-3.3 ^d	-8.5
<i>oil</i> : spot	0.3	-11.6
futures	3.3 ^e	3.3
<i>cmeq</i>	1.7	7.4
<i>reit</i>	-7.0	-7.0 ^f
<i>eval</i>	1.4	-1.4
<i>adj</i>	0.3	-5.9

^aWe assume that the CPI-linked bill has the same average payoff as T-bills, so that its expected excess return is zero by assumption. In practice, such a bill would likely yield lower returns than T-bills because of the inflation insurance it provides.

^bPrior to 1983, many commodities included in the GSCI spot index did not have traded futures. This series is constructed from a production-weighted average of futures contracts that are traded in each period.

^cThe CRB futures index became available only in 1986. In addition, its composition is somewhat different from that of the CRB spot index. For details on the components of these indexes, see various issues of *Commodity Research Bureau Futures Chart Service: Financial Section*, published weekly by Knight-Ridder Co.

^dGold futures began trading only in 1975.

^eOil futures began trading only in 1983.

^fThe REIT index data continue only through December 1991. Dividends are excluded from the index, an omission that has little effect on the correlations above, but an important effect on historical total returns. In Exhibit 10, we therefore added to the REIT index return the dividend yields from the Wilshire Associates REIT index (which begins only in 1978). Although starting somewhat later, the Wilshire REIT total return index performed much better than did the S&P subindex; its total excess returns are 7.8% over the 1978-1993 period and 2.6% over the 1983-1993 period.

assumptions for each hedging vehicle. First, we calculate target exposures and optimal hedge ratios under the assumption that the expected excess return on the real asset is zero.

Second, we try an expected excess return of 10 basis points. This latter assumption is not meant to capture realistic return expectations. Instead, it is used to identify the sensitivity of the optimal hedge ratio to changes in expected return. Because this sensitivity is

constant across different levels of expected return, we can use the change in the hedge ratio induced by a 10-basis point increase in expected return to compute easily the optimal hedge ratio for *any* expected return.

For example, suppose that the optimal hedge ratio is 10% with zero expected return and 12% with a 10-basis point expected return. Then an investor who expects an excess return of 50 basis points would have an optimal hedge ratio of $10\% + [(12\% - 10\%) \times 50 \text{ bp}/10 \text{ bp}] = 20\%$.

Finally, we try a third assumption about expected excess returns: that they are equal to whatever amount is needed to induce an investor to hold a real asset hedge of zero. In other words, the third assumption reports the expected return of an investor who chooses not to hedge at all. This return level is therefore a kind of threshold: Investors of similar risk tolerance who are more bullish than the reported expected return should be long; those more bearish should be short.

The results of these computations are reported in Exhibit 12.¹⁴ There are several points to note. First,

EXHIBIT 12

Optimal Real Asset Hedge Ratios for a Diversified Portfolio of Stocks and Bonds (as a fraction of preexisting portfolio value)

		Expected Return (basis points)	Target Exposure	Preexisting Exposure	Optimal Hedge Ratio
<i>unxpi</i>	1	0	0.00	-2.78	2.78
	2	10	9.49	-2.78	12.27
	3	-3	2.78	-2.78	0.00
<i>gsci</i>	1	0	0.00	-0.30	0.30
	2	10	0.03	-0.30	0.33
	3	-93	-0.30	-0.30	0.00
<i>oil</i>	1	0	0.00	-0.12	0.12
	2	10	0.01	-0.12	0.13
	3	-189	-0.12	-0.12	0.00
<i>reit</i>	1	0	0.00	0.26	-0.26
	2	10	0.02	0.26	-0.24
	3	107	0.26	0.26	0.00
<i>eval</i>	1	0	0.00	0.07	-0.07
	2	10	0.25	0.07	0.18
	3	3	0.07	0.07	0.00
<i>adj</i>	1	0	0.00	-0.22	0.22
	2	10	0.08	-0.22	0.30
	3	-26	-0.22	-0.22	0.00

unxpi's hedge ratios are extremely sensitive to changes in expected returns — a single basis point of additional expected return increases the optimal hedge ratio by almost 100% of the value of the preexisting portfolio. This is a result of the very low volatility of unexpected inflation, which makes optimizing investors very responsive to expected excess returns on hypothetical CPI-linked bills. A slightly negative excess return of -3 bp (to reflect the small amount of inflation insurance that CPI-linked bills would add to T-bills) is the threshold return that results in a hedge ratio of zero.

Second, note that an investor who has a zero hedge ratio expects strongly negative excess returns for both the GSCI and oil. This reflects the commodities' negative correlations with diversified portfolios as well as their relatively high volatilities.

Third, as one would expect, the hedge ratios for REITs are negative (unless expected excess returns are over 107 basis points). This implies that a short position in REITs is the optimal hedge when expected excess returns are under 107 basis points. Hedge ratios for the appraisal-based real estate returns are usually positive, although required returns for holding such hedges are considerable higher than those for the GSCI and oil.

CONCLUSION

In our examination of the hedging properties of real assets for a variety of diversified portfolios, we find that long exposures to a number of real assets — gold, the CRB index, commodity-linked equities, and, particularly, broad real estate indexes — provide relatively weak hedges for broadly diversified portfolios. Levered long positions in equity-based real estate and commodity stocks, even those of oil companies, do not reduce portfolio variance; on the contrary, such exposures actually tend to increase portfolio variance strongly.

By contrast, leveraged positions in commodities with a high energy component — such as oil and the GSCI — exhibit strong hedging properties, as such positions are able to reduce total portfolio variance significantly. The same is true for levered long positions in a hypothetical CPI-linked bond, which can be used to insulate stock and bond portfolios.

It appears that commodity hedges are, at least for stocks, more potent hedges than even CPI-linked bonds. The reason appears to be a negative correlation

between stock (and to some extent bond) returns and commodity prices, even when holding inflation fixed.

We combine these correlation results with several measures of expected return in order to calculate optimal hedge ratios for a fixed-weight portfolio of stocks and bonds. Our results suggest that, for this portfolio, diversification provides a powerful rationale for holding energy-weighted commodities.

The same cannot be said for gold, real estate, and equities in commodity and real estate businesses. With respect to a widely diversified portfolio of stocks and bonds, the rationale for holding gold, real estate, or industry equities must rely almost entirely on expected return relative to own volatility.

ENDNOTES

The author thanks Ernie Ankrim, Zvi Bodie, Jim Crimmins, Chris Hensel, Jay Light, André Pérol, Phil Scott, and Larry Siegel for useful comments and help in obtaining data, and the J. Aron Division of Goldman Sachs and the Department of Research at Harvard Business School for research support.

¹On the correlations of asset classes with inflation see Bodie [1976, 1979, 1982, 1983, 1990], Fama [1981, 1982], Gay and Manaster [1982], Geske and Roll [1983], Nelson [1976], and Stulz [1986].

²Important exceptions include Bodie [1976, 1979] and Ankrim and Hensel [1992, 1993].

³While real stock returns are negatively correlated with inflation at shorter horizons, the correlation appears to shrink toward zero at longer horizons. Empirical work suggests that it takes roughly five years before stock prices catch up with inflation shocks (see Boudoukh and Richardson [1993]). Nevertheless, many investors will find it optimal to hedge stocks against inflation shocks.

⁴For the purposes of hedging, one should use the *unexpected* percentage change in the value of the real asset to determine the exposure. Over reasonably short horizons, however, the variation over time in expected return is dwarfed by that of the actual return.

⁵See Froot and Pérol [1993] for a more formal derivation of what follows.

⁶For a description of the GSCI and its properties, see Ankrim and Hensel [1993] and Lummer and Siegel [1993].

⁷On the problems of measuring real estate returns, see, for example, Gyourko and Keim [1992].

⁸Yet another means of avoiding appraisal-based indexed is to use transaction-based indexes. Because the mix of properties that trade tends to vary with market conditions, transaction-based indexes are also problematic. Case and Shiller [1987] have developed a transaction-based index based on repeat sales in order to attenuate this problem of shifting mix. Their data seem to exhibit similar properties to *adj* (see the results reported by Goetzmann [1993]) but they include only residential real estate, and therefore the real estate is unlikely to be owned primarily by investors (as opposed to homeowners).

⁹We do not use the popular Russell-NCREIF Property Index because it begins only in 1978; its statistical properties appear very similar to those of the Evaluation Associates Index.

¹⁰Goetzmann and Ibbotson [1990] and Goetzmann [1993] find that both residential and commercial real estate have negative correlations with domestic bonds and smaller (probably insignificant) correlations with domestic stocks. Their results suggest that commercial

real estate returns are positively correlated with the S&P 500, but that residential and farmland appraisal-based returns are negatively correlated. The differences between our results and theirs may be partly attributable to differing sample periods and partly to differences between residential returns and those of other real estate classes.

¹¹Estimates for the same 1983–1993 sample period using spot commodity returns produce very similar results, and so are omitted. The CRB futures and spot indexes have similar hedging properties, but these indexes differ somewhat in their composition. See the notes to Exhibit 11.

¹²There is other evidence consistent with this view. For example, Ball and Mankiw [1992] argue that larger relative price shocks are associated with higher overall inflation; and Benabou [1992] finds inflation to be empirically associated with lower markups, a finding that can be interpreted as evidence that input price increases both cause inflation and are costly to industry.

¹³The estimated standard errors of the coefficient $\beta_{1,2}$ are correct only under the assumption that the hedge ratio for the initial real asset is known with certainty.

¹⁴Exhibit 12 assumes a risk tolerance coefficient of 0.5. If a smaller coefficient is used instead, the sensitivity of optimal hedge ratios to changes in expected returns is reduced proportionately.

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